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COMPLEX FUZZY EVOLUTION EQUATIONS

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Abstract. In this paper, we introduce the concept of complex fuzzy strongly continuous semigroup and investigate some of its properties. Using these notions, We establish the existence and uniqueness of mild solution to the complex fuzzy evolution equations. We also provide an illustrative example.

Keywords. Complex fuzzy evolution equations, Complex fuzzy sets, Fuzzy differential equations , Fuzzy sets, Mild solution, Semigroups.

1 Introduction

The theory of evolution equations has become an important area of investigation in recent years, stimulated by their numerous applications to problems from mechanics, electrical engineering, medicine, biology, ecology, etc.

Since D. Ramot introduced a new concept in the context of fuzzy sets theory (complex fuzzy set) and there are many examples of application namely solar activity, signal processing, etc (see [12]).

The Cauchy problem for complex fuzzy differential equations

$$\begin{cases} X'(t) = H(t, X(t)), & t \in [a, b], \\ X(a) = X_a, \end{cases}$$

is studied in [2] by Daria Karpenko, Robert A. Van Gorder and Abraham Kandel, where the mapping $H : [a, b] \times \mathbb{E} \longrightarrow \mathbb{E}$(fuzzy complex metric space) is Holder continuous and bounded.

In this work we study following problem:

$$\begin{cases} X'(t) = \mathcal{A}X(t) + H(t, X(t)), & t \in I \\ X(0) = X_0 \in \mathbb{E}. \end{cases} \quad (1)$$

Where $\mathcal{A}$ is the generator of a continuous semigroup $\{\pi(t), t \geq 0\}$ on $\mathbb{E}$ and $H : I \times \mathbb{E} \longrightarrow \mathbb{E}$ which we take to be continuous in both arguments and satisfies some conditions.

The paper is organized as follows. In "Preliminaries" section preliminaries and basic concepts on complex fuzzy sets and fuzzy derivative are given. In "Main Results" section we defined complex Fuzzy Semigroups and we proposed the complex fuzzy evolution equations. Additionally, an example is presented. Finally conclusions and future research scope of this article are drawn in last section, "Conclusion" section.

2 Preliminaries

First of all, we provide some notation and recall known results that are necessary in our study.

Let $\mathcal{P}_K(\mathbb{R}^n)$ denote the family of all nonempty compact convex subsets of $\mathbb{R}^n$ and define the addition and scalar multiplication in $\mathcal{P}_K(\mathbb{R}^n)$ as usual.

Let A and B be two nonempty bounded subsets of $\mathbb{R}^n$. The distance between A and B is defined by the Hausdorf metric,

$$d(A, B) = max \left\{ \sup_{a \in A} \inf_{b \in B} \|a - b\|, \sup_{b \in B} \inf_{a \in A} \|a - b\| \right\}$$

where $\|\|$ denotes the usual Euclidean norm in $\mathbb{R}^n$.

Then it is clear that $(\mathcal{P}_K(\mathbb{R}^n), d)$ becomes a complete and separable metric space (see [11]).
We denote by

$$E^n = \{u : \mathbb{R}^n \longrightarrow [0, 1] \;\mid\; \text{u satisfies (i)-(iv) below } \},$$

where

(i) u is normal i.e there exists an $x_0 \in \mathbb{R}^n$ such that $u(x_0) = 1$,

(ii) u is fuzzy convex,

(iii) u is upper semicontinuous,

(iv) $[u]^0 = cl\{x \in \mathbb{R}^n \mid u(x) > 0\}$ is compact.

For $0 < \alpha \leq 1$, denote $[u]^\alpha = \{t \in \mathbb{R}^n \mid u(t) \geq \alpha\}$. From (i)-(iv), it follows that the α-level set $[u]^\alpha \in \mathcal{P}_K(\mathbb{R}^n)$ for all $0 \leq \alpha \leq 1$.

According to Zadeh's extension principle, we have addition and scalar multiplication in fuzzy number space E^n are as follows

$$[u + v]^\alpha = [u]^\alpha + [v]^\alpha, \quad [ku]^\alpha = k[u]^\alpha$$

where $u, v \in E^n$, $k \in \mathbb{R}^n$ and $0 \leq \alpha \leq 1$.

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