Journal of Nonlinear Systems and Applications (2018) 52–56

http://www.watam.org/JNSA/

FLUCTUATIONS IN A DELAYED PEGI MODEL WITH ECONOMIC CHARACTERISTICS OF POPULATION

Abdelilah Kaddar1, Salah-Eddine Sahbani2 and Hamad Talibi Allaoui2 *†‡

Abstract. In this communication, we include the economic characteristics of population (employed, unemployed) in the economic growth Solow model. The resulting model is a system of three differential equations with three time delays. The principal objective of this model is to describe the Population-Economic Growth Interactions, hence its name PEGI model. The dynamic is studied in terms of the local stability and of the local Hopf bifurcation.

Keywords. Growth model; delay differential equations; Stability; Hopf bifurcation; periodic solutions.

1 Introduction

In this work, we propose the theoretical analysis of a solow economic growth model [24] with the economic characteristics of the concerned population (employed, unemployed). The resulting model is the following delayed differential system :

$$\begin{aligned} \frac{dK}{dt} &= sf(K(t-\tau_1), L(t)) - \delta K(t), \\ \frac{dL}{dt} &= \gamma pN(t)[1 - \frac{L(t-\tau_2)}{L_e}], \\ \frac{dN}{dt} &= \rho N(t)[1 - \frac{N(t-\tau_3)}{g(K(t-\tau_3), L(t-\tau_3))}], \\ L(t) &\leq N(t), \end{aligned} \tag{1}$$

where K is the capital stock, L is the size of the active population, pN is the number of unemployed (working age population), N is the total number of population, s is the saving rate, δ is the depreciation rate of capital stock, γ is the employment rate, ρ is the population growth rate, f is the production function, L_e is the effective labor demand [17], g is the carrying capacity, τ_1 is the Kalecki's time delay (1935, [20]); i.e. there is a time lag needed for new capital to be installed, τ_2 is the time needed to assess needs for labor force and the time taken for the recruitment of this labor force [13] and τ_3 is the lag between the birthdate and the puberty age (the time needed for a newborn to participate in the reproductive process) [11].

The topic of economic growth is fundamental to any discussion of economic issues. This concept is considered in the above system (see, (1)) by the first equation which describes the process of capital accumulation in the economy, since capital accumulation is the engine of growth thanks to saving and investment.

To study the relationship between economic growth and the dynamic of concerned population, we divide the last into three disjoint classes: active individuals L, and unemployed individuals pN. The evolution of the number of individuals in these compartments is given by the last two equations of the system (1).

The principal objective of the model (1) is to describe the mutual interactions between population and economic growth, hence its name PEGI model, i.e. Population-Economic Growth Interactions model. This interaction has attracted the attention of economic researchers who have analyzed empirically the economies of various countries. Their results can be divided into three main points of view: The first team states that population growth stimulates economic growth [23, 27, 26]. The second team think that population growth adversely affects economic growth [22] while a third team believes that population growth is a neutral factor in economic growth and is determined outside standard growth models[26].

This topic has also attracted the attention of mathematicians who developed some theoretical models. As examples and without limitation, we cite the following work:

The Solow-Swan economic growth model (1956, [24, 25]). This model is presented by the following ordinary differential equation:

$$\frac{dK}{dt} = sf(K(t), L_0 e^{nt}), \tag{2}$$

Here the author has assumed that the function f is with constant returns to scale, the growth rate of labor n and

*A. Kaddar is with Faculty of Law, Economics and Social Sciences-Salé, Mohammed V University in Rabat. E-mails: a.kaddar@yahoo.fr

†S. Sahbani and H. Talibi Allaoui are with Department of Mathematics, Faculty of Sciences, Chouaib Doukkali University. E-mail: sahbani8@gmail.com, $talibi_1$@$hotmail.fr$

‡Manuscript received May 2017; revised February 2018.