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SWITCHING RESTRICTIONS FOR STABILITY DESPITE SWITCHING DELAY: APPLICATION TO SWITCHED TRACKING TASKS IN PARKINSON'S DISEASE

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Abstract. Switched nonlinear systems with delay in the switching instant could be destabilized, despite stable dynamics in each mode, if the delay is long enough. We identify a restriction on the switching scheme to assure stability despite a finite delay in switching instant. The restriction partitions the state-space in a time-varying manner for a known switching delay, and converges to a steady-state partition that can be determined from the intersection of Lyapunov functions in each mode. We apply this technique to experimental data from a manual pursuit tracking task performed by 14 subjects with Parkinson's disease, and 10 control subjects. Each subject manually tracks a moving target through a joystick-controlled cursor, with sudden changes in the tracking dynamics. The tracking task can be modeled as a 3-mode switched system. By calculating the maximal time delay for each mode pair and for each subject, we obtain a measure of relative stability that can be compared across groups and across tasks. Using the derived stability measure, subjects with Parkinson's disease were shown to be relatively less stable than control subjects.

Keywords. Switched systems, switching delay, Parkinson's disease, nonlinear dynamics

1 Introduction

We consider stability of switched systems [1, 2, 3] with delay in the switching instant. While such systems are ubiquitous in engineered systems (such as those with communication delays or with humans in the loop) we focus here on application to a biological system – motor control in a switched manual pursuit tracking task in Parkinson's disease – for which stability despite switching delay provides a measure of robustness to sudden changes in task dynamics.

For switched systems with stable dynamics in each mode, existence of a common Lyapunov function proves stability under arbitrary switching [4, 5]. If a common Lyapunov function does not exist or cannot be found, stability can be assessed for a known switching signal [6, 7, 8] or assured by constraining switching to occur in certain regions of the state-space [9]. However, even if a switching signal is determined to result in stable behavior, a delay in the switching instant could result in performance degradation or even destabilization of the system.

In Parkinson's disease, a neurodegenerative disorder in which voluntary movement is impaired by a lack of dopamine in the brain, slowness in switching between multiple tasks (e.g., between reaching and balancing) may underlie the empirical observation that PD subjects have difficulty performing simultaneous movements. Indeed, the slowness in switching may be a contributing factor to the high prevalence of falls in Parkinson's disease as compared to the general population. In addition, cognitive inflexibility in Parkinson's disease [10, 11] may closely correspond to an inability to adapt to sudden change. When sudden changes occur in a dynamic context, as with manual pursuit tracking tasks, delay in motor response to sudden changes may be related to stability despite delay in switching. In this paper, we investigate the effect of delay on switching in Parkinson's disease through the use of a manual pursuit tracking task whose dynamics have sudden changes – that is, a tracking task that has hybrid dynamics.

A set of three motor tasks were designed to evaluate the ability of subjects to respond to sudden and unexpected changes in tracking dynamics. When switching between multiple tracking tasks, the subject must stabilize their error dynamics. In recent work [12], mode detection algorithms were used to determine that the length of time required for a subject to detect a sudden change in task dynamics (based solely on their tracking performance) was longer in subjects with Parkinson's disease as compared to normal subjects. In this paper, we take a slightly different approach. We instead evaluate the stability of each subjects' tracking dynamics, and determine an upper bound on the maximum delay in switching instant that each subject could tolerate without destabilization.

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