

EXISTENCE OF P -TH MEAN PSEUDO ALMOST AUTOMORPHIC SOLUTIONS TO SOME FUNCTIONAL STOCHASTIC INTEGRO-DIFFERENTIAL EVOLUTION EQUATIONS

Paul H. Bezandry ^{*†}

Abstract. In this paper, we investigate the existence of p -th mean pseudo almost automorphic solutions to some class of abstract functional stochastic integro-differential evolution equations in a real separable Hilbert space by the means of the well-known Schauder fixed point principle.

Keywords. Stochastic Integro-differentiation equation, p -th mean almost periodicity, p -th mean pseudo almost automorphy, Schauder fixed-point theorem, Wiener process

1 Introduction

The study of stochastic evolution equations is an exciting topic which brings together techniques from probability theory, functional analysis, and the theory of evolution equations. Most problems in a real life situation to which mathematical models are applicable are basically stochastic rather than deterministic. Stochastic evolution equations arise naturally in the mathematical modeling of phenomena in the natural sciences. Often, a mathematical model of such phenomena is subject to some random environment effects. Thus, stochastic differential equations reflect the reality better than differential equations.

The present paper investigates the existence of p -th mean pseudo almost automorphic solutions to some functional stochastic integro-differential evolution equations of the form

$$\begin{aligned} X'(t) = & A(t)X(t) + \int_{-\infty}^t C(t-u)G(u, X(u)) dW(u) \\ & + \int_{-\infty}^t B(t-u)F_2(u, X(u)) du + F_1(t, X(t)), \quad t \in \mathbb{R} \end{aligned} \quad (1)$$

Here, $p \geq 2$, $(A(t))_{t \in \mathbb{R}}$ is a family of densely defined closed linear operators satisfying Acquistapace and Terreni conditions; B and C are convolution-type kernels in $L^1(0, \infty)$ and $L^2(0, \infty)$; $F_1, F_2 : \mathbb{R} \times L^p(\Omega, \mathbb{H}) \rightarrow L^p(\Omega, \mathbb{H})$ and $G : \mathbb{R} \times L^p(\Omega; \mathbb{H}) \rightarrow L^p(\Omega; \mathbb{L}_2^0)$ are jointly continuous functions satisfying some additional conditions.

^{*}Department of Mathematics, Howard University, Washington, DC. E-mail: pbezandry@howard.edu

[†]Manuscript received January 5, 2011; accepted April 16, 2011.

The rapid development of the theory of integro-differential equations in infinite-dimensional spaces has been strongly promoted by the large number of applications in mechanics, electromagnetic theory, heat flow, nuclear reactor dynamics, and population dynamics.

The concept of pseudo almost automorphic function is a natural generalization of that of almost automorphic function, and the latter was first introduced by Bochner in [11,12] and is regarded as an important extension of the classical almost periodicity. In recent years, the existence of almost periodic, Stepanov automorphic, and almost automorphic solutions on stochastic evolution equations has been considerably investigated in lots of publications [6–10,13,20].

Recently in Bezandry [6,7], the case of semi-linear stochastic integro-differential equations of the form (1) in a real separable Hilbert space was considered. Under some suitable conditions it was proved that such equations have almost periodic and Stepanov-like almost periodic solutions. In a very recent paper [10], Bezandry and Diagana obtained an existence result on p -th mean pseudo almost automorphic mild solutions to some nonautonomous stochastic differential equation. In this paper, we use the same kind of assumptions as in Bezandry and Diagana [10] to study the existence of p -th mean pseudo almost automorphy of the solutions of problem (1). The techniques used in this work are taken from [18].

The rest of this paper is organized as follows: Section 2 is devoted to preliminaries facts related to the existence of an evolution family. Some preliminary results on intermediate spaces are also stated there. In Section 3, basic definitions and results on the concept of p -th mean pseudo almost periodicity and pseudo almost automorphy are given. In Section 4, we give some sufficient conditions for the existence of p -th mean pseudo almost automorphic solutions to some abstract functional integro-differential stochastic evolution equations (1).

2 Preliminaries

Let $(\mathbb{B}, \|\cdot\|)$ be a Banach space. If L is a linear operator on the Banach space \mathbb{B} , then $D(L)$, $\rho(L)$, $\sigma(L)$,